

**Exercise 12**

Differentiate the function.

$$h(x) = \ln(x + \sqrt{x^2 - 1})$$

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**Solution**

Take the derivative of the function.

$$\begin{aligned} h'(x) &= \frac{d}{dx} \left[ \ln(x + \sqrt{x^2 - 1}) \right] \\ &= \frac{1}{x + \sqrt{x^2 - 1}} \cdot \frac{d}{dx} (x + \sqrt{x^2 - 1}) \\ &= \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left[ \frac{d}{dx}(x) + \frac{d}{dx} \sqrt{x^2 - 1} \right] \\ &= \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left[ (1) + \frac{1}{2}(x^2 - 1)^{-1/2} \cdot \frac{d}{dx}(x^2 - 1) \right] \\ &= \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left[ (1) + \frac{1}{2}(x^2 - 1)^{-1/2} \cdot (2x) \right] \\ &= \frac{1}{x + \sqrt{x^2 - 1}} \left( 1 + \frac{x}{\sqrt{x^2 - 1}} \right) \\ &= \frac{1}{x + \sqrt{x^2 - 1}} \left( \frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}} \right) \\ &= \frac{1}{\sqrt{x^2 - 1}} \end{aligned}$$